MHD Casson fluid with heat transfer in a liquid film over unsteady stretching plate

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ABSTRACT

In this research, we analyze heat transfer of MHD boundary layer flow of Casson fluid. Strong nonlinear ordinary differential equations are solved numerical using Shooting method with fourth order Runge-Kutta (RK4) integration technique. Variations of interesting different parameters on the velocity are showed graphically.

1. Introduction

The characteristics and analysis of flow and heat transfer of thin films have attracted the attention of many researchers. This is refer to their multi-applications in engineering such as food stuff processing, reactor fluidization, wire and fiber coating, cooling of metallic plates, drawing of a polymer sheet, aerodynamic extrusion of plastic sheets, continuous casting, rolling, annealing, and tinning of copper wires. In the extrusion process, this understanding is crucial for maintenance of the surface quality of extradite. All coating process requires a smooth glossy surface for the best product appearance and properties like as low friction, strength and transparency. As the quality of product in the extrusion processes depends considerably on the film flow and heat transfer characteristics of a thin liquid over a stretching plate, investigated and analysis of momentum and heat transfer in such processes is essential.

Lawrence (1970) was the first among others researchers to consider the steady two-dimensional boundary layer flow driven by a stretching of a sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Liu and Megahed (2012) investigated the effects of variable heat flux and internal heat generation on the flow and heat transfer in a thin liquid film on a horizontal stretching plate in the presence of thermal radiation,

similarity transformations are used to reduce the governing equations to a set of coupled nonlinear ordinary differential equations. Subhas et al. (2009) analyzed mathematical model of MHD flow and heat transfer to a laminar liquid film from a horizontal sheet, the flow of a thin fluid film and subsequent heat transfer from the stretching surface is investigated with the aid of similarity transformation, the transformation enables to transform the unsteady boundary layer equations to a system of non-linear ordinary differential equations. Baag et al. (2016) analyzed the magneto-hydrodynamic fluid flow on a stretching plate in a porous medium. The effects of magnetic field and permeability of the medium on the flow field are investigated. They have considered flow of a conducting viscous fluid through porous sheet using Darcy model refer to a variable magnetic field. Hayat et al. (2008) studied heat transfer effects on the steady flow of a generalized Burgers' fluid induced by a sudden pull of eccentric rotating disks, and showed the effect of parameter on velocity, temperature, force, and torque exerted by the fluid on one of the disks. Recently, Yan et al. (2015) examined two elastic heat transfer tube bundles and comprehensive comparison on vibration and heat transfer on them. Also, Boltenko et al. (2015) investigated of pressure drop and heat transfer in an annular channel with heat transfer intensifiers. The effect of heat transfer enhancement by dimpled surface heat exchanger in thermoelectric generator studied numerically by (Yiping et al., 2016). Manjunatha et al. (2015) discussed the effect of radiation on flow and heat transfer of MHD dusty fluid over a stretching cylinder embedded in a porous medium in presence of heat source. Finally, Ahmed investigated MHD viscous Casson fluid flow

and heat transfer second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation (Megahed, 2015). Recently Zeeshan et al. (2016a) discussed the effect of magnetic dipole and heat transfer analysis on Jeffery fluid flow over a stretching sheet with suction/injection. Also, Zeeshan et al. (2016b) studied the effect of magnetic dipole on viscous ferro-fluid past a stretching surface with thermal radiation.

According the above descriptions, the main objective of this study is to apply Shooting method with fourth order Runge-Kutta (RK4) integration technique to find the approximate solution of nonlinear differential equations governing the problem of flow and heat transfer of MHD boundary layer flow of Casson fluid. The main difference and novelty of this work is solving the energy equation and find the thermal boundary layer by the present analytical method.

2. Mathematical model

The MHD boundary layer flow over a flat plate is governed by the continuity and the Navier-Stokes equations for an incompressible viscous fluid. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the stretching sheet. The induced magnetic field is neglected. The resulting boundary-layer equations are (Eqs. 1-3):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) - \sigma\frac{B^2(x)}{\rho}u - \frac{v}{K}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = K\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

where, u and v are the velocity components in the x and y directions, respectively. Also v, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid. Parameter of the Casson fluid is $\beta = \frac{\mu_B\sqrt{2\pi}}{P}$, T is the temperature and K is the thermal diffusivity of the fluid and a transverse magnetic field of uniform strength $B(x)$ is equal to $B(x) = B_0 x^{n-\frac{1}{2}}$.

The boundary conditions are given below in Eqs. 4 and 5:

$$t = 0:\ u(t,x,y) = ax,\ v(t,x,y) = 0,$$
$$T = T_w(x,t)\ at\ y = 0,$$
$$u(t,x,y) = v(t,x,y) = T(t,x,y) = 0, \text{at } y > 0 \tag{4}$$
$$t > 0:\ u = ax, v = 0,\ T = T_w(x,t), \text{at } y = 0$$
$$u = 0,\ \ T = T_\infty\ \ as\ \ y \to \infty \tag{5}$$

here, $T_w = T_\infty + \frac{c\xi^{-\frac{3}{2}}x^2}{2v}$, and T_0 is a heating or cooling temperatures. To solve the problem, momentum and energy equations are firstly non-dimensional zed by introducing the following dimensionless variables: Define ψ as the stream function where $u =$

$\frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. Applying the following transformation (Eq. 6):

$$\psi = \sqrt{av\xi}\ xf(\eta,\xi);$$
$$\eta = \sqrt{\frac{a}{v\xi}}\ y;\ \ \ \xi = 1 - exp(-\tau);$$
$$\tau = a\ t,$$
$$T = \left[T_\infty + \frac{c\xi^{-\frac{3}{2}}x^2}{2v}\right]\theta(\eta) \tag{6}$$

$M = \frac{\sigma B^2(x)}{a\rho},\ \ \lambda = \frac{v}{aK}$, where λ is porosity parameter.

Putting Eq. 6 into Eqs. $1-3$. Eq. 1 is automatically satisfied, Eq. 2 and Eq. 3 becomes

$$\left(1 + \frac{1}{\beta}\right)\frac{\partial^3 f}{\partial\eta^3} + \frac{1}{2}(1-\xi)\eta\frac{\partial^2 f}{\partial\eta^2} + \xi\left(f\frac{\partial^2 f}{\partial\eta^2} - \left(\frac{\partial f}{\partial\eta}\right)^2 - M\left(\frac{\partial f}{\partial\eta}\right) -$$
$$\lambda\left(\frac{\partial f}{\partial\eta}\right)\right) = \xi(1-\xi)\frac{\partial^2 f}{\partial\eta\partial\xi} \tag{7}$$

$$\frac{1}{2}\frac{\partial\theta}{\partial\eta}\eta(\xi - 1) + \frac{3}{2}\theta(\xi^3 - \xi^2) + 2\xi\frac{\partial f}{\partial\eta}\ \theta - \xi f\frac{\partial\theta}{\partial\eta} =$$
$$\frac{1}{Pr}\frac{\partial^2 f}{\partial\eta^2} \tag{8}$$

where, $Pr = \frac{\vartheta}{K}$. The boundary conditions (4) and (5), become (Eqs. 9 and 10):

$$(0,\xi) = 0,\ \ \frac{\partial f}{\partial\eta}(0,\xi) = 1,$$
$$\theta(0,\xi) = 1\ \ \ at\ \ \ \ \eta = 0 \tag{9}$$
$$\frac{\partial f}{\partial\eta}(\infty,\xi) \to 0,\ \ \ \theta(\infty,\xi) \to 0,\ at\ \ \eta \to \infty \tag{10}$$

We consider $\xi = 1$, where Eq. 7 and Eq. 8 becomes (Eqs. 11 and 12):

$$\left(1 + \frac{1}{\beta}\right)\frac{d^3 f}{d\eta^3} + f\left(\frac{d^2 f}{d\eta^2}\right) - \left(\frac{df}{d\eta}\right)^2 - M\left(\frac{df}{d\eta}\right) - \lambda\left(\frac{df}{d\eta}\right) = 0 \tag{11}$$
$$\frac{1}{Pr}\frac{d^2\theta}{d\eta^2} - 2\frac{df}{d\eta}\ \theta + f\frac{d\theta}{d\eta} = 0 \tag{12}$$

The boundary condition, (9) and (10), becomes (Eqs. 13 and 14):

$$f(0) = 0,\ \frac{\partial f}{\partial\eta}(0) = 1,\ \ \theta(0) = 1 \tag{13}$$
$$\frac{\partial f}{\partial\eta}(\infty) \to 0,\ \ \ \theta(\infty) \to 0 \tag{14}$$

3. Numerical solution

The dimensionless for velocity and temperature Eqs. 11 and 12 with the boundary conditions (13) and (14) have been solved numerically by shooting method with fourth order Runge-Kutta (RK4) integration technique. Firstly we reduce the original ODEs into a system of first order ODEs by setting:

$$\omega_1 = f, \omega_2 = f', \omega_3 = f'', \omega_4 = \theta, \omega_5 = \theta'$$

which gives Eq. 15 and the corresponding initial conditions are Eq. 16.

To solve Eq. 15 with Eq. 16 as an initial value problem we must need the values for φ_1 and φ but no such values are given.

$$\begin{bmatrix} \omega_1{}' \\ \omega_2{}' \\ \omega_3{}' \\ \omega_4{}' \\ \omega_5{}' \end{bmatrix} = \begin{bmatrix} \omega_2 \\ \omega_3 \\ \frac{-\omega_1\omega_3 + \omega_2{}^2 + M\omega_2 + \lambda\omega_2}{\left(1+\frac{1}{\beta}\right)} \\ \omega_5 \\ Pr(2\omega_2\,\omega_4 - \omega_1\omega_5) \end{bmatrix} \qquad (15)$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \varphi_1 \\ 1 \\ \varphi_2 \end{bmatrix} \qquad (16)$$

The initial guess values for $F''(0)$ and $\theta'(0)$ are chosen and the fourth order Runge–Kutta integration scheme is applied to obtain the solution. The maximum value of $\eta \to \infty$, to each group of parameters is determined when the values of unknown boundary conditions at $\eta = 0$ do not change to a successful loop with error less than 10^{-5}.

4. Discussion and Results

In this section, one can obtain the solution of the problem numerically. Appropriate similarity transformation is used to transform and change the governing partial differential equations of fluid flow and heat transfer equation into a system of non-linear ordinary differential equations. The lasted boundary value problem is solved by the efficient shooting method (Figs. 1 and 2).
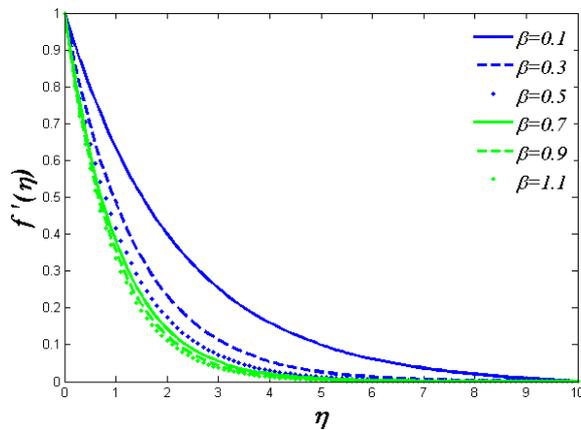
Fig. 1: Variation of β on $f'(\eta)$
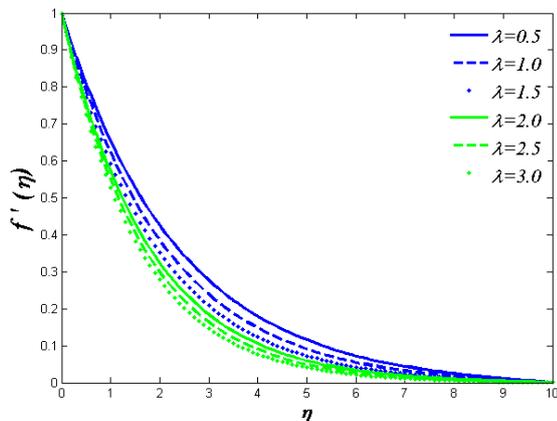
Fig. 2: Variation of λ on $f'(\eta)$

In fact, we discuss and analyze the different interesting physical parameters, such as fluid parameter β, Hartmann number M, porosity parameter λ, and Prandtl number Pr.

Figs. 1 and 2 Show the effect of Casson fluid β and porosity parameter λ, for different values of parameters. It show graphically that the magnitude of velocity and boundary layer thickness decreases with an increase in fluid parameter β and λ. It is notice that the Hartmann number M, showed in Fig. 3 is decrease coefficient of boundary layer fluid flow. It is evident from Fig. 4, that the Prandtl number is a decrease coefficient of boundary value and $\theta'(\eta)$.
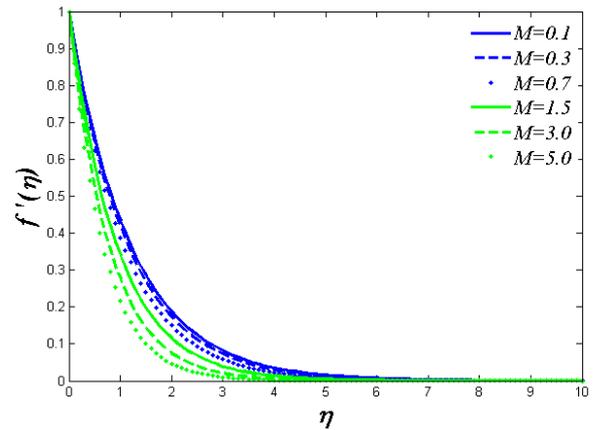
Fig. 3: Variation of M on $f'(\eta)$
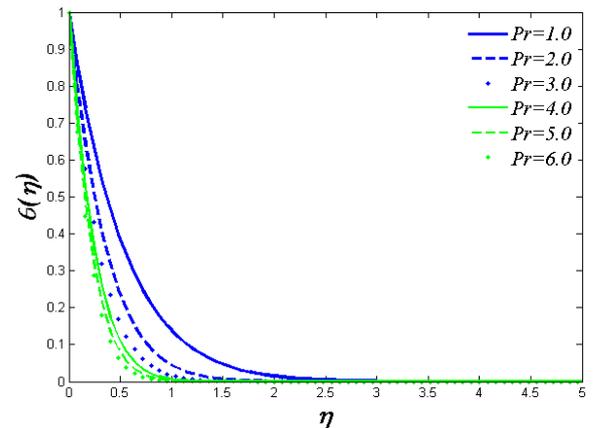
Fig. 4: Variation of Pr on $\theta(\eta)$

5. Conclusion

In this article, MHD boundary layer flow of Casson fluid is examined with heat transfer. Simultaneous effects of energy equation are also considered. The governing nonlinear coupled differential equations are solved numerically with the help of shooting method. The impact of all the penitent parameters is discussed and illustrated with the help of graphs.

References

Baag S, Mishra SR, Dash GC, and Acharya MR (2016). Entropy generation analysis for viscoelastic MHD flow over a stretching sheet embedded in a porous medium. Ain Shams Engineering Journal. https://doi.org/10.1016/j.asej.2015. 10.017

Boltenko EA, Varava AN, Dedov AV, Zakharenkov AV, Komov AT, and Malakhovskii SA (2015). Investigation of heat transfer and pressure drop in an annular channel with heat transfer intensifiers. Thermal Engineering, 62(3): 177-182.

Hayat T, Maqbool K, and Khan M (2008). Hall and heat transfer effects on the steady flow of a generalized Burgers' fluid induced by a sudden pull of eccentric rotating disks. Nonlinear Dynamics, 51(1-2): 267-276.

Lawrence CJ (1970). Flow past a stretching plate. Zeitschrift für angewandte Mathematik und Physik ZAMP, 21(4): 645-647.

Liu I and Megahed AM (2012). Homotopy perturbation method for thin film flow and heat transfer over an unsteady stretching sheet with internal heating and variable heat flux. Journal of Applied Mathematics Volume 2012 (2012), Article ID 418527, 12 pages. https://doi.org/10.1155/2012/418527

Manjunatha PT, Gireesha BJ, and Prasannakumara BC (2015). Effect of radiation on flow and heat transfer of MHD dusty fluid over a stretching cylinder embedded in a porous medium in presence of heat source. International Journal of Applied and Computational Mathematics: 1-18.

Megahed AM (2015). MHD viscous Casson fluid flow and heat transfer with second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation. The European Physical Journal Plus, 130(4): 1-17.

Subhas MA, Mahesha N, and Tawade J (2009). Heat transfer in a liquid film over an unsteady stretching surface with viscous dissipation in presence of external magnetic field. Applied Mathematical Modelling, 33(8): 3430-3441.

Yan K, Ge PQ, and Zhai Q (2015). A comprehensive comparison on vibration and heat transfer of two elastic heat transfer tube bundles. Journal of Central South University, 22(1): 377-385.

Yiping W, Li S, Yang X, Deng Y and Su C (2016). Numerical and experimental investigation for heat transfer enhancement by dimpled surface heat exchanger in thermoelectric generator. Journal of Electronic Materials, 45(3): 1792-1802.

Zeeshan A and Majeed A (2016a). Heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction/injection and magnetic dipole effect. Alexandria Engineering Journal, 55(3): 2171-2181.

Zeeshan A, Majeed A and Ellahi R (2016b). Effect of magnetic dipole on viscous ferro-fluid past a stretching surface with thermal radiation. Journal of Molecular Liquids, 215: 549-554.